Nonlinear and Nonequilibrium Dynamics in Geomaterials

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Research conducted during the past decade, much of it conducted at Los Alamos National Laboratory (LANL), has pioneered a new direction in the study of nonlinear-elastic dynamical behavior in materials. It has been found that certain materials display surprisingly large dynamical nonlinearities, with effects due to conditioning and memory appearing already at strains smaller than parts per million. Furthermore, these strains relax over very long durations, a nonequilibrium phenomenon termed slow dynamics. The existence of this behavior over a wide spectrum of complex materials promises a new era in understanding and characterizing their mechanical behavior: geomaterials are the prototype for these systems, encompassing apparently unrelated consolidated and unconsolidated solids, also including sintered and damaged metals and some ceramics. These materials are characterized by a fabric of elastically soft features ("bonds") within a hard matrix ("grains"). While the bond system is responsible for the static and dynamic elastic properties, it exists only within a small fraction of the total volume, less than 1%. The precise microphysics of the nonlinear behavior remains an unsolved problem, though there are some clues as to the scales and processes leading to the bulk behavior.

Until very recently, a useful classification of the nonlinear dynamical phenomena did not exist. Our recent experimental and theoretical work [1] has now established that below a threshold strain, the materials are elastically linear, while above this threshold they display (reversible) nonlinear softening—the regime of fast nonlinear dynamics (FND)—until a second threshold is reached beyond which conditioning and memory effects become apparent. If a material is driven well above the second threshold it relaxes back to the original state over a long period of time (as long as hours). This is slow dynamics (SD). Dynamics near the second strain threshold is a combination of FND and SD, which we term mixed dynamics (MD). Before our latest work, MD was considered to be a purely nonlinear elastic phenomenon, but now we know it to be a combination of dynamics and relaxation.

Our major conclusions are as follows. Below a threshold strain of 10^{-8}–10^{-7} for two

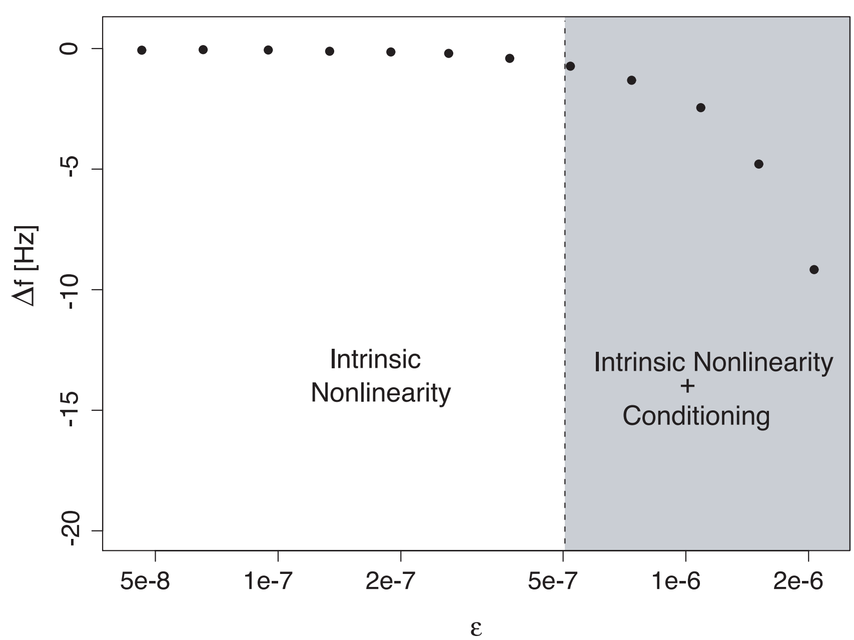

Figure 1— ***Resonance frequency shift versus strain in Berea sandstone. The regime of intrinsic material nonlinearity is shown in white, the regime of combined nonlinear and nonequilibrium behavior in gray.***

sandstone samples there was no discernible dependence of the resonance frequency on the strain—the materials behaved linearly to better than 1 part in 10^4. Above this level of strain, the materials displayed a reversible softening of the resonance frequency with strain, in excellent qualitative agreement with the quadratic prediction of classical nonlinear Landau theory up to a point where memory and conditioning effects became apparent. In detailing and characterizing the onset and nature of the nonlinearity, our results very substantially improve on previous work. In addition we also showed that, up to the conditioning threshold, the dynamical behavior is accurately captured by a phenomenological macroscopic model incorporating a (softening) Duffing nonlinearity and linear losses. Beyond the conditioning threshold, the simultaneous presence of nonlinearity and nonequilibrium dynamics complicates the characterization of dynamical behavior; in the absence of a separation of these effects, the data cannot be interpreted to support the existence of nonclassical behavior—as was erroneously claimed in previous work. It was previously thought that a characteristic feature of FND materials was a linear dependence of the frequency shift on the driving amplitude, however, we have shown that below the second strain threshold, the frequency shift is quadratically dependent on the drive amplitude. The interpretation of results at higher strains is complicated; we aim to carry out a new class of experiments designed to address this particular issue.

Slow dynamics, a striking instance of structural memory, occurs in all materials that display FND. Slow Dynamics was discovered at LANL and the French Petroleum Institute during the mid-90s. In the materials so far investigated, after the initial source excitation has died away (100–1000 microsec), the resonance frequency (and therefore the modulus) takes 10^3 to 10^4 seconds to recover to its original value. This recovery can be continuously tracked by using a small-amplitude wave probe, operating either in frequency or time domain. Excellent long-term thermal stability is necessary for such experiments. We hope to begin them soon in Theoretical Division at LANL using an acoustically isolated chamber within an internal room in the building. The chamber has been tested for thermal stability and found to be satisfactory.

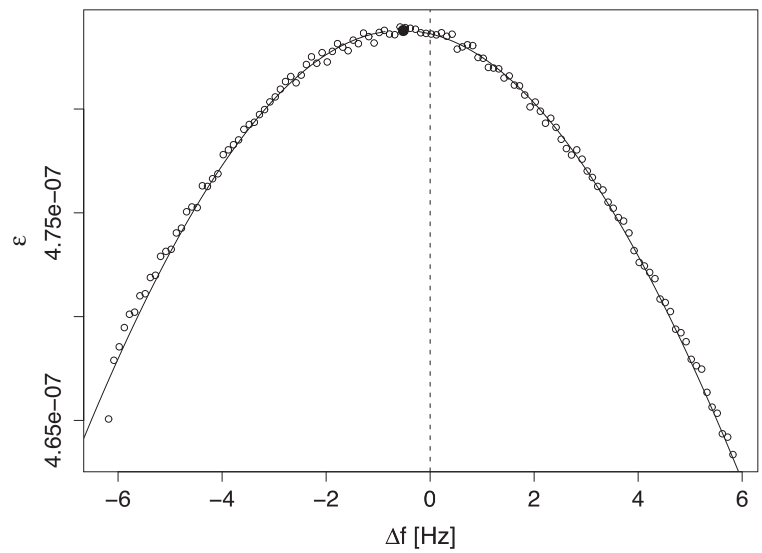

Figure 2— ***Detail of the resonance curve response peak for Berea sandstone showing the nonlinear frequency softening (left-shift of the peak from the zero-point). The solid curve is the theoretical prediction from Ref. [1].***

The primary theoretical idea behind our investigation is to view the linkage between grains as due to a complex set of "bonds" connecting one grain to another, the bonds being treated as nonlinear springs and the grains being treated as passive masses. Mesoscopic dynamical properties follow from averaging over the individual bonds, thus all of these properties can be encoded in a distribution function of effective individual bond spring constants. Upon external driving or delivery of a thermal or mechanical impulse, the distribution changes in such a way that the modulus decreases; this can be pictured as being due either to material softening (due to dislocations, for instance) or selective "breaking" of individual bonds (e.g., due to brittleness of stiff bonds). SD follows immediately as well: once the driving is removed, the bond system returns to the original thermal equilibrium state via a generalized Kramers barrier-crossing process (e.g., dislocation hopping or bond healing).

[1] J.A. Tencate, D. Pasqualini, S. Habib, K. Heitmann, D. Higdon, and P.A. Johnson, *Phys. Rev. Lett.* **93**, 065501 (2004).